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Generalized Scheme for Splitting Arbitrary 2-Qubit State with Three 2-Qubit Entangled States

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Abstract A generalized tripartite scheme is proposed for splitting an arbitrary 2-qubit pure state by utilizing three 2-qubit entangled states as quantum channels. In the scheme the splitter averagely partitions its unknown 2-qubit state between two agents and either agent can recover the unknown state in a probabilistic manner with the other agent's assistance. 32 unitary operations used possibly and the total success probability of the scheme are worked out. Moreover, some discussions are made, especially on the relation between the success probability and the entanglements in the quantum channels.

Keywords Generalized scheme · Quantum state splitting · Arbitrary 2-qubit state · 2-qubit entangled state

1 Introduction

With the theory of quantum mechanics in the field of information, many interesting developments have been produced in last decades, such as quantum key distribution [1, 2], quantum teleportation [3–5], quantum dense coding [6–8], quantum secret sharing (QSS) [9–11], etc. QSS is the quantum counterpart of classical secret sharing [12, 13] in quantum scenarios. This generalization from classical information processing to quantum information processing was first presented by Hillery, Bŭzek and Berthiaume (HBB) in 1999 [9]. By far it has attracted much attention and many QSS schemes have already been proposed [14–29]. The QSS of quantum information is conventionally termed as quantum state splitting (QSTS) to differentiate from the QSS of classical message. About QSTS, so far different

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quantum channels consisting of various entangled states have been explored, such as GHZ states [30–32], W states [33, 34], Graph states [35], Bell states [36–38], etc. Very recently, people focus more attentions on the non-maximally entangled states as quantum channels [39–42]. For examples, in 2006 Gordon and Rigolin [40] first proposed a generalized QSTS scheme for sharing an arbitrary unknown single-qubit state by using a non-maximally entangled GHZ state and a pair of non-maximally entangled Bell state as quantum channels, respectively. Their work showed that it is indeed possible to construct QSTS schemes using partially entangled states. In 2007 Wang et al. [42] presented first a three-party scheme for probabilistically implementing quantum state sharing of an arbitrary unknown two-qubit state and then sketched it to multi-party case, where non-maximally entangled three-qubit states are employed as quantum channels. In this paper, we will put forward a generalized tripartite scheme for splitting an arbitrary 2-qubit state by using three 2-qubit non-maximally entangled states as quantum channels. Besides, we will reveal the relation between the success probability and the entanglements in the quantum channels. Incidentally, Deng et al. [36, 37] have recently presented two QSTS schemes for splitting an arbitrary two-qubit state by swapping entanglements in two-qubit Bell states. Their schemes are convenient in application for its several distinct advantages [38]. Our present work is essentially a generalization of Deng et al.'s work [36, 37].

This paper is organized as follows. In Sect. 2, a generalized tripartite scheme for probabilistically splitting an arbitrarily unknown 2-qubit pure state is amply presented by utilizing three 2-qubit non-maximally entangled states as quantum channels. In Sect. 3, some discussions and a brief summary are made.

Before introducing our schemes, we first define some useful notations. Let the four Pauli operators be written as $\sigma^0 = |0\rangle\langle 0| + |1\rangle\langle 1|$, $\sigma^1 = |0\rangle\langle 0| - |1\rangle\langle 1|$, $\sigma^2 = |1\rangle\langle 0| + |0\rangle\langle 1|$ and $\sigma^3 = |1\rangle\langle 0| - |0\rangle\langle 1|$. With these definitions, four Bell states can be represented in a compact form as

$$|\Psi^k\rangle_{ij} = \sigma_j^k \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{ij} \quad (k = 0, 1, 2, 3). \tag{1}$$

2 Our Generalized Tripartite QSTS Scheme

Now let us amply introduce our tripartite QSTS scheme for splitting an arbitrary 2-qubit quantum state via three 2-qubit non-maximally entangled states. The schematic demonstration is illustrated in Fig. 1. The scheme contains three legitimate parties. Suppose they are Alice, Bob and Charlie. Alice is the boss (also the splitter) in the scheme, Bob and Charlie are her two agents. Alice has an unknown quantum state which inhabits her qubit pair (x, x'). The state can be written as

$$|\mu\rangle_{xx'} = (\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle)_{xx'}, \tag{2}$$

where α, β, γ and δ are unknown complex numbers and satisfy $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Essentially, the four coefficients contain the information of the unknown state. Moreover, Alice, Bob and Charlie hold the qubit pairs (1,2), (1′,3′) and (2′,3), respectively. See Fig. 1(a). These three qubit pairs form the quantum channels linking the three parties and their states read

$$|\varphi^i\rangle_{ii'} = h_i|00\rangle_{ii'} + p_i|11\rangle_{ii'} \quad (i = 1, 2, 3), \tag{3}$$

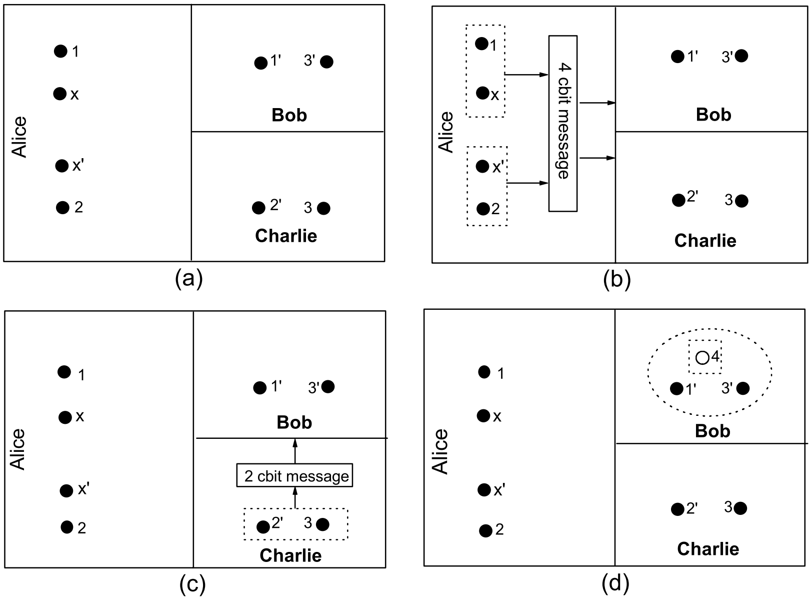

Fig. 1 Partition of an arbitrary 2-qubit quantum information among 2 recipients by using three 2-qubit entangled states. The *solid dots* represent qubits. The *little hollow circle* stands for the auxiliary qubit. The *dotted rectangles* characterize Bell-state measurements. The *dotted ellipse* symbolizes a three-qubit unitary operation. The *dotted square* means a single-qubit measurement. See text for more details

where h_i and p_i are real and positive, $h_i \geq p_i$, and $h_i^2 + p_i^2 = 1$. Alice wants to split her unknown state between Bob and Charlie by utilizing the quantum channels. About the split, her requirement is that neither agent can get the quantum state solely unless they cooperate. To achieve her goal, Alice first measures her two qubit pairs $(x,1)$ and $(x',2)$ in Bell-state bases, respectively (see Fig. 1(b)). Note that, the joint state of the quantum system composed of the six qubits x, x', 1, $1'$, 2 and $2'$ can be written as

$$
\begin{aligned}
&|\mu\rangle_{xx'} \otimes |\varphi^1\rangle_{11'} \otimes |\varphi^2\rangle_{22'} \\
&\quad = \frac{1}{2}[(|\Psi^0\rangle_{x1} + |\Psi^1\rangle_{x1}\sigma_{1'}^1)(|\Psi^0\rangle_{x'2} + |\Psi^1\rangle_{x'2}\sigma_{2'}^1) \\
&\qquad \times (\alpha h_1 h_2|00\rangle + \beta h_1 p_2|01\rangle + \gamma p_1 h_2|10\rangle + \delta p_1 p_2|11\rangle)_{1'2'} \\
&\qquad + (|\Psi^0\rangle_{x1} + |\Psi^1\rangle_{x1}\sigma_{1'}^1)(|\Psi^2\rangle_{x'2}\sigma_{2'}^2 + |\Psi^3\rangle_{x'2}\sigma_{2'}^3) \\
&\qquad \times (\alpha h_1 p_2|00\rangle + \beta h_1 h_2|01\rangle + \gamma p_1 p_2|10\rangle + \delta p_1 h_2|11\rangle)_{1'2'} \\
&\qquad + (|\Psi^2\rangle_{x1}\sigma_{1'}^2 + |\Psi^3\rangle_{x1}\sigma_{1'}^3)(|\Psi^0\rangle_{x'2} + |\Psi^1\rangle_{x'2}\sigma_{2'}^1) \\
&\qquad \times (\alpha p_1 h_2|00\rangle + \beta p_1 p_2|01\rangle + \gamma h_1 h_2|10\rangle + \delta h_1 p_2|11\rangle)_{1'2'} \\
&\qquad + (|\Psi^2\rangle_{x1}\sigma_{1'}^2 + |\Psi^3\rangle_{x1}\sigma_{1'}^3)(|\Psi^2\rangle_{x'2}\sigma_{2'}^2 + |\Psi^3\rangle_{x'2}\sigma_{2'}^3) \\
&\qquad \times (\alpha p_1 p_2|00\rangle + \beta p_1 h_2|01\rangle + \gamma h_1 p_2|10\rangle + \delta h_1 h_2|11\rangle)_{1'2'}].
\end{aligned} \tag{4}
$$

Apparently, Alice's measurements will induce the correlation of qubits $1'$ and $2'$ in different locations. Different measurement results correspond to different correlations. BTW, in physics the correlation comes essentially from entanglement swapping. To let Bob and Charlie know the resultant correlation exactly, Alice should publicly inform her two agents of the measurement results. In terms of their prior agreement, Alice publishes the classical bits (cbits) "$\hat{k}\hat{l}$" to notify her measurement result $|\Psi^k\rangle_{x1}|\Psi^l\rangle_{x'2}$ $(k, l = 0, 1, 2, 3)$, where the symbols $\hat{0} \equiv 00$, $\hat{1} \equiv 01$, $\hat{2} \equiv 10$ and $\hat{3} \equiv 11$ are used. Alice may get any one of the 16 measurement results. Without loss of generality, we assume she gets $|\Psi^0\rangle_{x1}|\Psi^0\rangle_{x'2}$ as an example. In this case, Alice should publicly announce the cbits "0000". From the (4) one can see that, in this instance Bob's qubit $1'$ and Charlie's qubit $2'$ correlate each other with their state being

$$|\omega^{00}\rangle_{1'2'} = \frac{1}{2}(\alpha h_1 h_2|00\rangle + \beta h_1 p_2|01\rangle + \gamma p_1 h_2|10\rangle + \delta p_1 p_2|11\rangle)_{1'2'}. \tag{5}$$

Obviously, one can see that the four coefficients now in the unknown state $|\mu\rangle$ appear in $|\omega^{00}\rangle_{1'2'}$. This indicates that $|\omega^{00}\rangle_{1'2'}$ contains the information about the unknown state $|\mu\rangle$. Alternatively, the original quantum information has already been split into the qubits in Bob and Charlie's sites separately.

Bob and Charlie can recover Alice's original quantum state in a probabilistic manner via their mutual assistance. In other words, either Bob or Charlie can conclusively restore the quantum information with the other's help. Due to symmetry, we now assume that Bob finally gets the original unknown state. To realize this, Charlie performs a Bell-state measurement on his qubits $2'$ and 3 (see Fig. 1(c)). Since the following equation holds,

$$\begin{aligned}
&|\omega^{00}\rangle_{1'2'} \otimes |\varphi^3\rangle_{33'} \\
&\quad = \frac{1}{2\sqrt{2}}\big[(|\Psi^0\rangle_{2'3} + |\Psi^1\rangle_{2'3}\sigma^1_{3'})(\alpha h_1 h_2 h_3|00\rangle \\
&\qquad + \beta h_1 p_2 p_3|01\rangle + \gamma p_1 h_2 h_3|10\rangle + \delta p_1 p_2 p_3|11\rangle)_{1'3'} + (|\Psi^2\rangle_{2'3}\sigma^2_{3'} + |\Psi^3\rangle_{2'3}\sigma^3_{3'}) \\
&\qquad \times (\alpha h_1 h_2 p_3|00\rangle + \beta h_1 p_2 h_3|01\rangle + \gamma p_1 h_2 p_3|10\rangle + \delta p_1 p_2 h_3|11\rangle)_{1'3'}\big],
\end{aligned} \tag{6}$$

one can easily see that, Charlie's measurement will lead to the following collapse:

$$\begin{aligned}
|\Psi^0\rangle_{2'3} \quad &\Rightarrow \quad |\omega^{000}\rangle_{1'3'} = \frac{1}{2\sqrt{2}}(\alpha h_1 h_2 h_3|00\rangle + \beta h_1 p_2 p_3|01\rangle \\
&\qquad + \gamma p_1 h_2 h_3|10\rangle + \delta p_1 p_2 p_3|11\rangle)_{1'3'},
\end{aligned} \tag{7}$$

$$\begin{aligned}
|\Psi^1\rangle_{2'3} \quad &\Rightarrow \quad |\omega^{001}\rangle_{1'3'} = \frac{1}{2\sqrt{2}}\sigma^1_{3'}(\alpha h_1 h_2 h_3|00\rangle + \beta h_1 p_2 p_3|01\rangle \\
&\qquad + \gamma p_1 h_2 h_3|10\rangle + \delta p_1 p_2 p_3|11\rangle)_{1'3'},
\end{aligned} \tag{8}$$

$$\begin{aligned}
|\Psi^2\rangle_{2'3} \quad &\Rightarrow \quad |\omega^{002}\rangle_{1'3'} = \frac{1}{2\sqrt{2}}\sigma^2_{3'}(\alpha h_1 h_2 p_3|00\rangle + \beta h_1 p_2 h_3|01\rangle \\
&\qquad + \gamma p_1 h_2 p_3|10\rangle + \delta p_1 p_2 h_3|11\rangle)_{1'3'},
\end{aligned} \tag{9}$$

$$\begin{aligned}
|\Psi^3\rangle_{2'3} \quad &\Rightarrow \quad |\omega^{003}\rangle_{1'3'} = \frac{1}{2\sqrt{2}}\sigma^3_{3'}(\alpha h_1 h_2 p_3|00\rangle + \beta h_1 p_2 h_3|01\rangle \\
&\qquad + \gamma p_1 h_2 p_3|10\rangle + \delta p_1 p_2 h_3|11\rangle)_{1'3'}.
\end{aligned} \tag{10}$$

If Charlie gets the measurement outcome $|\Psi^0\rangle_{2'3}$ (or $|\Psi^1\rangle_{2'3}$, $|\Psi^2\rangle_{2'3}$, $|\Psi^3\rangle_{2'3}$), he publishes the cbits "00" (or "01", "10", "11"). Once receiving this message, Bob introduces an auxiliary qubit 4 in the initial state $|0\rangle$ and carries out a collective unitary transformation U (or $U\sigma_{3'}^1$, $U'\sigma_{3'}^2$, $U'\sigma_{3'}^3$) on his qubits $1'$, $3'$ and 4 (see Fig. 1(d)). Complementarily, the unitary operators U and U' take the following forms

$$
\begin{aligned}
U = {} & \frac{p_1p_2p_3}{h_1h_2h_3}(|000\rangle\langle 000| - |001\rangle\langle 001|) + \frac{p_1}{h_1}(|010\rangle\langle 010| - |011\rangle\langle 011|) \\
& + \frac{p_2p_3}{h_2h_3}(|100\rangle\langle 100| - |101\rangle\langle 101|) + |110\rangle\langle 110| - |111\rangle\langle 111| \\
& + \sqrt{1-\left(\frac{p_1p_2p_3}{h_1h_2h_3}\right)^2}(|000\rangle\langle 001| + |001\rangle\langle 000|) \\
& + \sqrt{1-\left(\frac{p_1}{h_1}\right)^2}(|010\rangle\langle 011| + |011\rangle\langle 010|) \\
& + \sqrt{1-\left(\frac{p_2p_3}{h_2h_3}\right)^2}(|100\rangle\langle 101| + |101\rangle\langle 100|),
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
U' = {} & \frac{p_1p_2}{h_1h_2}(|000\rangle\langle 000| - |001\rangle\langle 001|) + \frac{p_1p_3}{h_1h_3}(|010\rangle\langle 010| - |011\rangle\langle 011|) \\
& + \frac{p_2}{h_2}(|100\rangle\langle 100| - |101\rangle\langle 101|) + \frac{p_3}{h_3}(|110\rangle\langle 110| - |111\rangle\langle 111|) \\
& + \sqrt{1-\left(\frac{p_1p_2}{h_1h_2}\right)^2}(|000\rangle\langle 001| + |001\rangle\langle 000|) \\
& + \sqrt{1-\left(\frac{p_1p_3}{h_1h_3}\right)^2}(|010\rangle\langle 011| + |011\rangle\langle 010|) \\
& + \sqrt{1-\left(\frac{p_2}{h_2}\right)^2}(|100\rangle\langle 101| + |101\rangle\langle 100|) \\
& + \sqrt{1-\left(\frac{p_3}{h_3}\right)^2}(|110\rangle\langle 111| + |111\rangle\langle 110|).
\end{aligned} \tag{12}
$$

The operations U, $U\sigma_{3'}^1$, $U'\sigma_{3'}^2$ and $U'\sigma_{3'}^3$ respectively cause the following transformations

$$
\begin{aligned}
U|\omega^{000}\rangle_{1'3'}|0\rangle_4 &= U\sigma_{3'}^1|\omega^{001}\rangle_{1'3'}|0\rangle_4 \\
&= \frac{1}{2\sqrt{2}}[p_1p_2p_3|\mu\rangle_{1'3'}|0\rangle_4 + (\alpha|\overline{00}\rangle + \beta|\overline{01}\rangle + \gamma|\overline{10}\rangle)_{1'3'}|1\rangle_4],
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
U'\sigma_{3'}^2|\omega^{002}\rangle_{1'3'}|0\rangle_4 &= U'\sigma_{3'}^3|\omega^{003}\rangle_{1'3'}|0\rangle_4 \\
&= \frac{1}{2\sqrt{2}}[p_1p_2p_3|\mu\rangle_{1'3'}|0\rangle_4 + (\alpha|\widetilde{00}\rangle + \beta|\widetilde{01}\rangle + \gamma|\widetilde{10}\rangle + \delta|\widetilde{11}\rangle)_{1'3'}|1\rangle_4],
\end{aligned} \tag{14}
$$

where

$$|\overline{00}\rangle = \sqrt{(h_1h_2h_3)^2 - (p_1p_2p_3)^2}|00\rangle, \qquad |\overline{01}\rangle = p_2p_3\sqrt{h_1^2 - p_1^2}|01\rangle,$$
$$|\overline{10}\rangle = p_1\sqrt{(h_2h_3)^2-(p_2p_3)^2}|10\rangle, \qquad |\widetilde{00}\rangle = p_3\sqrt{(h_1h_2)^2-(p_1p_2)^2}|00\rangle,$$
$$|\widetilde{01}\rangle = p_2\sqrt{(h_1h_3)^2-(p_1p_3)^2}|01\rangle, \qquad |\widetilde{10}\rangle = p_1p_3\sqrt{h_2^2-p_2^2}|10\rangle, \quad \text{and}$$
$$|\widetilde{11}\rangle = p_1p_2\sqrt{h_3^2-p_3^2}|11\rangle$$

(same hereafter). After the unitary operation, Bob performs a measurement on his qubit 4 under the basis of $\{|0\rangle, |1\rangle\}$. From the (13) and (14) one can see that, if the measurement outcome is $|0\rangle_4$, then Bob successfully gets $|\mu\rangle_{1'3'}$. Otherwise, the outcome is $|1\rangle_4$ and in this case Bob can not reconstruct the original state. This means the QSTS process fails in the latter case.

Up to now, we only consider Alice's measurement result is $|\Psi^0\rangle_{x1}|\Psi^0\rangle_{x'2}$. If Alice gets any other result, the secret state can also be probabilistically recovered on Bob's qubits $1'$ and $3'$ via the similar processes as before. This can be summarily seen from the following equation,

$$\begin{aligned}
&|\mu\rangle_{xx'} \otimes |\varphi^1\rangle_{11'} \otimes |\varphi^2\rangle_{22'} \otimes |\varphi^3\rangle_{33'}|0\rangle_4 \\
&= \frac{1}{2\sqrt{2}}\{(|\Psi^0\rangle_{x1} + |\Psi^1\rangle_{x1}\sigma^1_{1'})(|\Psi^0\rangle_{x'2}|\Psi^0\rangle_{2'3} + |\Psi^1\rangle_{x'2}|\Psi^1\rangle_{2'3} + |\Psi^0\rangle_{x'2}|\Psi^1\rangle_{2'3}\sigma^1_{3'} \\
&\quad + |\Psi^1\rangle_{x'2}|\Psi^0\rangle_{2'3}\sigma^1_{3'})U^\dagger[(p_1p_2p_3|\mu\rangle_{13}|0\rangle_4 + (\alpha|\overline{00}\rangle + \beta|\overline{01}\rangle + \gamma|\overline{10}\rangle)_{1'3'}|1\rangle_4] \\
&\quad + (|\Psi^2\rangle_{x1} - |\Psi^3\rangle_{x1}\sigma^1_{1'})(|\Psi^0\rangle_{x'2}|\Psi^0\rangle_{2'3} + |\Psi^1\rangle_{x'2}|\Psi^1\rangle_{2'3} + |\Psi^0\rangle_{x'2}|\Psi^1\rangle_{2'3}\sigma^1_{3'} \\
&\quad + |\Psi^1\rangle_{x'2}|\Psi^0\rangle_{2'3}\sigma^1_{3'})U^\dagger[(p_1p_2p_3\sigma^2_{1'}|\mu\rangle_{13}|0\rangle_4 + (\gamma|\overline{00}\rangle + \delta|\overline{01}\rangle + \alpha|\overline{10}\rangle)_{1'3'}|1\rangle_4] \\
&\quad + (|\Psi^0\rangle_{x1} + |\Psi^1\rangle_{x1}\sigma^1_{1'})(|\Psi^2\rangle_{x'2}|\Psi^0\rangle_{2'3} - |\Psi^3\rangle_{x'2}|\Psi^1\rangle_{2'3} + |\Psi^2\rangle_{x'2}|\Psi^1\rangle_{2'3}\sigma^1_{3'} \\
&\quad - |\Psi^3\rangle_{x'2}|\Psi^0\rangle_{2'3}\sigma^1_{3'})U^\dagger[(p_1p_2p_3\sigma^2_{3'}|\mu\rangle_{13}|0\rangle_4 + (\beta|\overline{00}\rangle + \alpha|\overline{01}\rangle + \delta|\overline{10}\rangle)_{1'3'}|1\rangle_4] \\
&\quad + (|\Psi^2\rangle_{x1} - |\Psi^3\rangle_{x1}\sigma^1_{1'})(|\Psi^2\rangle_{x'2}|\Psi^0\rangle_{2'3} - |\Psi^3\rangle_{x'2}|\Psi^1\rangle_{2'3} + |\Psi^2\rangle_{x'2}|\Psi^1\rangle_{2'3}\sigma^1_{3'} \\
&\quad - |\Psi^3\rangle_{x'2}|\Psi^0\rangle_{2'3}\sigma^1_{3'})U^\dagger[(p_1p_2p_3\sigma^2_{1'}\sigma^2_{3'}|\mu\rangle_{13}|0\rangle_4 + (\delta|\overline{00}\rangle + \gamma|\overline{01}\rangle + \beta|\overline{10}\rangle)_{1'3'}|1\rangle_4] \\
&\quad + (|\Psi^0\rangle_{x1} + |\Psi^1\rangle_{x1}\sigma^1_{1'})(|\Psi^0\rangle_{x'2}|\Psi^2\rangle_{2'3}\sigma^2_{3'} + |\Psi^1\rangle_{x'2}|\Psi^3\rangle_{2'3}\sigma^2_{3'} + |\Psi^0\rangle_{x'2}|\Psi^3\rangle_{2'3}\sigma^3_{3'} \\
&\quad + |\Psi^1\rangle_{x'2}|\Psi^2\rangle_{2'3}\sigma^3_{3'})U'^\dagger[(p_1p_2p_3|\mu\rangle_{13}|0\rangle_4 \\
&\quad + (\alpha|\widetilde{00}\rangle + \beta|\widetilde{01}\rangle + \gamma|\widetilde{10}\rangle + \delta|\widetilde{11}\rangle)_{1'3'}|1\rangle_4] \\
&\quad + (|\Psi^2\rangle_{x1} - |\Psi^3\rangle_{x1}\sigma^1_{1'})(|\Psi^0\rangle_{x'2}|\Psi^2\rangle_{1'3'}\sigma^2_{3'} + |\Psi^1\rangle_{x'2}|\Psi^3\rangle_{1'3'}\sigma^2_{3'} + |\Psi^0\rangle_{x'2}|\Psi^3\rangle_{1'3'}\sigma^3_{3'} \\
&\quad + |\Psi^1\rangle_{x'2}|\Psi^2\rangle_{1'3'}\sigma^3_{3'})U'^\dagger[(p_1p_2p_3\sigma^2_{1'}|\mu\rangle_{13}|0\rangle_4 \\
&\quad + (\gamma|\widetilde{00}\rangle + \delta|\widetilde{01}\rangle + \alpha|\widetilde{10}\rangle + \beta|\widetilde{11}\rangle)_{1'3'}|1\rangle_4] \\
&\quad + (|\Psi^0\rangle_{x1} + |\Psi^1\rangle_{x1}\sigma^1_{1'})(|\Psi^2\rangle_{x'2}|\Psi^2\rangle_{2'3}\sigma^2_{3'} - |\Psi^3\rangle_{x'2}|\Psi^3\rangle_{2'3}\sigma^2_{3'} + |\Psi^2\rangle_{x'2}|\Psi^3\rangle_{2'3}\sigma^3_{3'} \\
&\quad - |\Psi^3\rangle_{x'2}|\Psi^2\rangle_{2'3}\sigma^3_{3'})U'^\dagger[(p_1p_2p_3\sigma^2_{3'}|\mu\rangle_{13}|0\rangle_4 \\
&\quad + (\beta|\widetilde{00}\rangle + \alpha|\widetilde{01}\rangle + \delta|\widetilde{10}\rangle + \gamma|\widetilde{11}\rangle)_{1'3'}|1\rangle_4]
\end{aligned}$$

$$+ (|\Psi^2\rangle_{x1} - |\Psi^3\rangle_{x1}\sigma^1_{1'})(|\Psi^2\rangle_{x'2}|\Psi^2\rangle_{2'3}\sigma^2_{3'} - |\Psi^3\rangle_{x'2}|\Psi^3\rangle_{2'3}\sigma^2_{3'} + |\Psi^2\rangle_{x'2}|\Psi^3\rangle_{2'3}\sigma^3_{3'}$$
$$- |\Psi^3\rangle_{x'2}|\Psi^2\rangle_{2'3}\sigma^3_{3'})U'^{\dagger}[(p_1p_2p_3\sigma^2_{1'}\sigma^2_{3'}|\mu\rangle_{13}|0\rangle_4$$
$$+ (\delta|\widetilde{00}\rangle + \gamma|\widetilde{01}\rangle + \beta|\widetilde{10}\rangle + \alpha|\widetilde{11}\rangle)_{1'3'}|1\rangle_4]\}. \tag{15}$$

From the equation one can see that, corresponding to Alice and Charlie's 64 different measurement results Bob should choose appropriately one of 32 different unitary operations. For another example, assume Alice and Charlie get $|\Psi^3\rangle_{x1}|\Psi^2\rangle_{x'2}|\Psi^2\rangle_{2'3}$ or $|\Psi^3\rangle_{x1}|\Psi^3\rangle_{x'2}|\Psi^3\rangle_{2'3}$. In these two cases, first Bob needs to carry out the unitary operation $\sigma^2_{1'}\sigma^2_{3'}U'\sigma^1_{1'}\sigma^2_{3'}$ on his qubits $1'$, $3'$ and 4. Then he performs a single-qubit measurement on the qubit 4. If he gets $|0\rangle_4$, the secret state is successfully reconstructed. Otherwise, the scheme fails. So far we have already detailed our QSTS scheme.

3 Discussions and Summary

Now let us make some brief discussions on this generalized tripartite QSTS scheme. (i) The quantum channels used in our scheme are general. In this paper the quantum channels are three 2-qubit entangled states, which we write in two-term forms (see (3)). Apparently, they seem to be special entangled states, for any 2-qubit state can be intuitively written in the form of

$$|\varphi^i\rangle_{mn} = a_i|00\rangle_{mn} + b_i|01\rangle_{mn} + c_i|10\rangle_{mn} + d_i|11\rangle_{mn}, \tag{16}$$

where a_i, b_i, c_i and d_i are complex and satisfy $|a_i|^2 + |b_i|^2 + |c_i|^2 + |d_i|^2 = 1$. As matter of fact, in terms of Schmidt decomposition [43, 44], any two-qubit entangled state taken the form as (16) can be reexpressed as

$$|\varphi^i\rangle_{mn} = h'_i|u^i v^i\rangle_{mn} + p'_i|u^i_\perp v^i_\perp\rangle_{mn}, \tag{17}$$

where h'_i and p'_i are real and positive, $\{|u^i\rangle, |u^i_\perp\rangle\}$ and $\{|v^i\rangle, |v^i_\perp\rangle\}$ are two sets of Schmidt bases. (ii) The success probability of our scheme is determined by the entanglements in the quantum channels. From the (15) one can see that, corresponding to any measurement result $|\Psi^k\rangle_{x1}|\Psi^l\rangle_{x'2}|\Psi^m\rangle_{2'3}$ $(k, l, m = 0, 1, 2, 3)$ the success probability of our scheme is $\frac{1}{8}(p_1p_2p_3)^2$. Thus the total success probability is $8(p_1p_2p_3)^2$. Furthermore, the coefficient p_i $(i = 1, 2, 3)$ correspond to the entropy of the quantum channel [45], i.e.

$$E_i = -(1 - p_i^2)\log_2(1 - p_i^2) - p_i^2\log_2 p_i^2. \tag{18}$$

Besides, when $0 < p_i < \frac{1}{\sqrt{2}}$, (18) is a monofonic function. Therefore, the success probability is actually the function of the entropies of the quantum channels. Note that, this entropy represents in essence the inherent property (i.e., entanglements) in the quantum channels. Consequently, the success probability is determined by the inherent entanglements in quantum channels. (iii) Our scheme is a generalization of Deng et al.'s scheme [37]. In our scheme the coefficients p_1, p_2 and p_3 range from 0 to $\frac{1}{\sqrt{2}}$. When they are chosen to be $\frac{1}{\sqrt{2}}$, our scheme reduces to Deng et al.'s scheme [37]. In this case, the auxiliary qubit 4 is needless and the total success probability is $8(p_1p_2p_3)^2 = 1$. As a consequence, the probabilistic scheme has changed into a deterministic one.

To summarize, in this paper we have proposed a three-party QSTS scheme for probabilistically splitting an unknown 2-qubit state via three 2-qubit partially entangled states. In the scheme, the splitter (Alice) averagely splits her unknown 2-qubit state between her two agents by performing two Bell-state measurements. Afterwards, either agent can recover the unknown state in a probabilistic manner by introducing an auxiliary qubit and performing an appropriate unitary operation on its qubits, provided that the other agent assists by executing a Bell-state measurement and informing of the measurement result beforehand. Moreover, we have made some discussions on the scheme including the quantum channels, the success probability and the scheme reduction.

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References

1. Bennett, C.H., Brassard, G.: In: Proceedings of IEEE International Conference on Computers, Systems, and Signal Processing, p. 175. Bangalore, India. IEEE Press, New York (1984)
2. Ekert, A.K.: Phys. Rev. Lett. **67**, 661 (1991)
3. Bennett, C.H., Brassard, G., Crepeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Phys. Rev. Lett. **70**, 1895 (1993)
4. Deng, F.G., et al.: Phys. Rev. A **72**, 022338 (2005)
5. Zhang, Z.J., et al.: Phys. Lett. A **372**, 28 (2007)
6. Bennett, C.H., Wiesner, S.J.: Phys. Rev. Lett. **69**, 2881 (1992)
7. Pati, A.K., Parashar, P., Agrawal, P.: Phys. Rev. A **72**, 012329 (2005)
8. Yang, W.X., Gong, Z.X.: J. Phys. B **40**, 1245 (2007)
9. Hillery, M., Bužek, V., Berthiaume, A.: Phys. Rev. A **59**, 1829 (1999)
10. Karlsson, A., Koashi, M., Imoto, N.: Phys. Rev. A **59**, 162 (1999)
11. Xiao, L., et al.: Phys. Rev. A **69**, 052307 (2004)
12. Blakley, G.R.: In: Proceedings of the American Federation of Information Processing 1979 National Computer Conference, pp. 313–317. American Federation of Information Processing, Arlington, VA (1979)
13. Shamir, A.: Commun. ACM **22**, 612 (1979)
14. Cleve, R., Gottesman, D., Lo, H.K.: Phys. Rev. Lett. **83**, 648 (1999)
15. Tittel, W., Zbinden, H., Gisin, N.: Phys. Rev. A **63**, 042301 (2001)
16. Chau, H.F.: Phys. Rev. A **66**, 060302 (2002)
17. Hsu, L.Y.: Phys. Rev. A **68**, 022306 (2003)
18. Zhang, Z.J., et al.: Phys. Rev. A **71**, 044301 (2005)
19. Zhang, Z.J., et al.: Phys. Rev. A **72**, 022303 (2005)
20. Yan, F.L., Gao, T.: Phys. Rev. A **72**, 012304 (2005)
21. Yang, W.X., et al.: Phys. Rev. A **72**, 062108 (2005)
22. Yang, W.X., et al.: Mod. Phys. Lett. B **22**, 1243 (2008)
23. Hsu, L.Y., Li, C.M.: Phys. Rev. A **71**, 022321 (2005)
24. Zhang, Z.J.: Physica A **361**, 233 (2006)
25. Zhang, Z.J.: Opt. Commun. **261**, 199 (2006)
26. Han, L.F., Liu, Y.M., Shi, S.H., Zhang, Z.J.: Phys. Lett. A **361**, 24 (2007)
27. Zhou, P., Li, X.H., Liang, Y.J., Deng, F.G., Zhou, H.Y.: Physica A **381**, 164 (2007)
28. Gaertner, S., Kurtsiefer, C., Bourennane, M., Weinfurter, H.: Phys. Rev. Lett. **98**, 020503 (2007)
29. Wang, Z.Y., Yuan, H., Shi, S.H., Zhang, Z.J.: Eur. Phys. J. D **41**, 371 (2007)
30. Yu, Y.F., Feng, J., Zhan, M.S.: Phys. Rev. A **66**, 052310 (2002)
31. Yan, F.L., Wang, D.: Phys. Lett. A **316**, 297 (2003)
32. Yu, Y.F., Feng, J., Zhan, M.S.: Phys. Rev. A **68**, 024303 (2003)
33. Zheng, S.B.: Phys. Rev. A **74**, 054303 (2006)
34. Zhang, Z.J., Cheung, C.Y.: J. Phys. B **41**, 015503 (2008)

35. Markham, D., Sanders, B.C.: Phys. Rev. A **78**, 042309 (2008)
36. Deng, F.G., Li, X.H., Li, C.Y., Zhou, H.Y.: Phys. Rev. A **72**, 044301 (2005)
37. Deng, F.G., Li, X.H., Li, C.Y., Zhou, P., Zhou, H.Y.: Eur. Phys. J. D **39**, 459 (2006)
38. Yuan, H., Liu, Y.M., Zhang, W., Zhang, Z.J., Cheung, C.Y.: J. Phys. B **41**, 145506 (2008)
39. Agrawal, P., Pati, A.K.: Phys. Lett. A **305**, 12 (2002)
40. Gordon, G., Rigolin, G.: Phys. Rev. A **73**, 062316 (2006)
41. Li, W.L., Li, C.F., Guo, G.C.: Phys. Rev. A **61**, 034301 (2000)
42. Wang, Z.Y., Liu, Y.M., Wang, D., Zhang, Z.J.: Opt. Commun. **276**, 322 (2007)
43. Ekert, A., Knight, P.: Am. J. Phys. **63**, 415 (1995)
44. Pati, A.K.: Phys. Lett. A **278**, 118 (2000)
45. Bennett, C.H., Bernstein, H.J., Popescu, S., Schumacher, B.: Phys. Rev. A **53**, 2046 (1996)